Unit 5 Progress Check: FRQ Part A

1. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Let f be a twice-differentiable function such that $f'(1) = 0$. The second derivative of f is given by $f''(x) = x^2 \cos(x^2 + \pi)$ for $-1 \leq x \leq 3$.

(a) On what open intervals contained in $-1 < x < 3$ is the graph of f concave up? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(b) Does f have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(c) Use the Mean Value Theorem on the closed interval $[-1, 1]$ to show that $f'(-1)$ cannot equal 2.5.

Please respond on separate paper, following directions from your teacher.

(d) Does the graph of f have a point of inflection at $x = 0$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Part A

A maximum of 1 out of 2 points is earned for one correct interval with reason and no incorrect intervals.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ intervals
- ☐ reason

Solution:

The graph of f is concave up on $(1.253,\ 2.170\ (\text{or } 2.171))$ and $(2.802,\ 3)$ because $f''(x)$ is positive on those intervals.

Part B

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Note: Sign charts are a useful tool to investigate and summarize the behavior of a function. By itself a sign chart for $f'(x)$ or $f''(x)$ is not a sufficient response for a justification.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- ☐ answer
- ☐ justification

Solution:

Because $f'(1) = 0$ and $f''(1) = -0.540302 < 0$, by the Second Derivative Test f has a relative maximum at $x = 1$.

Part C

The first point requires use of average rate of change of f' on $[-1, 1]$.

One point is earned for using MVT to conclude that there would be a number c in the interval $(-1, 1)$ such that $f''(c) = -1.25$. Two points are earned for showing that this is not possible.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The student response accurately includes all three of the criteria below.

- ☐ considers average rate of change
- ☐ conclusion using Mean Value Theorem

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Solution:

If $f'(-1) = 2.5$, then the average rate of change of f' on the closed interval $[-1, 1]$ would be $\frac{f'(1)-f'(-1)}{1-(-1)} = \frac{0-2.5}{2} = -1.25$.

Since f' is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$, by the Mean Value Theorem there would be a number c in the interval $(-1, 1)$ such that $f''(c) = -1.25$.

However, since $f''(x) = x^2 \cos(x^2 + \pi)$, it follows that $-1 \leq f''(x) \leq 1$ on the interval $(-1, 1)$. Therefore, there is no number c in the interval such that $f''(c) = -1.25$.

Thus, it is not possible that $f'(-1) = 2.5$.

Part D

Note: Sign charts are a useful tool to investigate and summarize the behavior of a function. By itself a sign chart for $f'(x)$ or $f''(x)$ is not a sufficient response for a justification.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ answer
- ☐ reason

Solution:

Because $f''(x)$ does not change sign at $x = 0$, the graph of f does not have a point of inflection there.