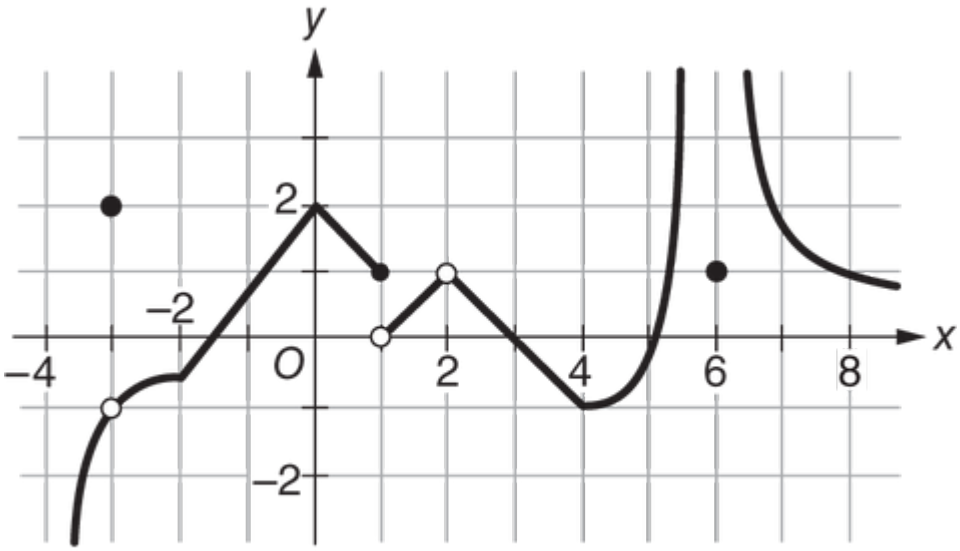
Unit 1 Progress Check: FRQ Part A

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f

x	-0.2	-0.1	-0.01	-0.001	0.001	0.01	0.1	0.2
$g(x)$	1.975	1.988	1.997	1.999	2.001	2.003	2.012	2.025

The graph of the function f is shown in the xy-plane above. f has a discontinuity due to a vertical asymptote at $x = 6$.

Let g be a function that is continuous and increasing for all x. Values of $g(x)$ at selected values

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of x are shown in the table above.

(a) Using the graph of f and the table for g, estimate $\lim_{x \to 0} (f(x) - g(x))$.

Please respond on separate paper, following directions from your teacher.

(b) For each of the values $a = 1$, $a = 2$, $a = 4$, and $a = 6$, find $\lim_{x \to a} f(x)$ or explain why $\lim_{x \to a} f(x)$ does not exist. Use correct limit notation in your answers.

Please respond on separate paper, following directions from your teacher.

(c) Is f continuous at $x = -3$? Using correct notation, justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) Write a difference quotient that best approximates the instantaneous rate of change of g at $x = 0$.

Please respond on separate paper, following directions from your teacher.

Part A

Both limits must be correct to earn the answer point. The response may still earn the points with use of equals rather than approximately.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

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The student response accurately includes both of the criteria below.

- ☐ estimate for $\lim_{x \to 0} g(x)$
- ☐ answer

Solution:

$\lim_{x \to 0} f(x) = 2$ and $\lim_{x \to 0} g(x) \approx 2$

$\lim_{x \to 0} (f(x) - g(x)) = \lim_{x \to 0} f(x) - \lim_{x \to 0} g(x) \approx 2 - 2 = 0$

Part B

At most 2 out of 3 points if mathematical notation for any limit is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

- ☐ does not exist at $a = 1$ with reason
- ☐ values at $a = 2$ and $a = 4$
- ☐ does not exist at $a = 6$ with reason

Solution:

$\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = 0$

$\lim_{x \to 1} f(x)$ does not exist because

$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

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$\lim_{x \to 2} f(x) = 1$

$\lim_{x \to 4} f(x) = -1$

$\lim_{x \to 6} f(x)$ does not exist because

$\lim_{x \to 6^-} f(x) = \infty$ and $\lim_{x \to 6^+} f(x) = \infty$.

Part C

At most 1 out of 3 points if mathematical notation for limit is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

- ☐ limit notation
- ☐ limit value
- ☐ answer with justification

Solution:

$\lim_{x \to -3} f(x) = -1$ and $f(-3) = 2$

No, f is not continuous at $x = -3$ because $\lim_{x \to -3} f(x) \neq f(-3)$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

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0	1 ✔

The student response accurately includes a correct difference quotient.

Solution:

$\frac{g(0.001)-g(-0.001)}{0.001-(-0.001)} = \frac{2.001-1.999}{0.001-(-0.001)}$

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2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Let f be the function defined by $f(x) = \frac{ax^2+bx+2}{2x^2-8}$, where a and b are constants. The graph of f has a horizontal asymptote at $y = 3$, and f has a removable discontinuity at $x = 2$.

(a) Show that $a = 6$ and $b = -13$.

Please respond on separate paper, following directions from your teacher.

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(b) To make f continuous at $x = 2$, $f(2)$ should be defined as what value? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(c) At what value of x does f have a discontinuity due to a vertical asymptote? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Part A

The last point is earned for work that concludes with the correct values for both a, and b.

At most 3 out of 4 points if mathematical notation for limits is missing or incorrect.

Students do not have to show the factorization to earn the point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The student response accurately includes all four of the criteria below.

- ☐ recognizes $\lim_{x \to \infty} f(x) = 3$
- ☐ shows that $\lim_{x \to \infty} f(x) = \frac{a}{2}$
- ☐ sets $ax^2 + bx + 2 = 0$ at $x = 2$
- ☐ verification

Solution:

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$\lim_{x\to\infty} f(x) = 3$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2\left(a+\frac{b}{x}+\frac{2}{x^2}\right)}{x^2\left(2-\frac{8}{x^2}\right)} = \lim_{x\to\infty} \frac{a+\frac{b}{x}+\frac{2}{x^2}}{2-\frac{8}{x^2}} = \frac{a}{2}$$

$\Rightarrow\ a = 6$

Since f has a removable discontinuity at $x = 2$ and the denominator $2x^2 - 8$ equals 0 at $x = 2$, the numerator must also equal 0 at $x = 2$.

$$0 = 6 \cdot 2^2 + 2b + 2 = 26 + 2b$$
$\Rightarrow\ b = -13$

Part B

At most 1 out of 2 points if mathematical notation for any limit is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ uses $\lim_{x\to 2} f(x)$
- ☐ answer with justification

Solution:

f will be continuous at $x = 2$. if $f(2) = \lim_{x\to 2} f(x)$.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{6x^2-13x+2}{2x^2-8} = \lim_{x\to 2} \frac{(6x-1)(x-2)}{2(x+2)(x-2)}$$
$$= \lim_{x\to 2} \frac{6x-1}{2(x+2)} = \frac{11}{8}$$

$f(2)$ should be defined as $\frac{11}{8}$ to make f continuous at $x = 2$.

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Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response .

0	1 ✔

The student response accurately indicates $x = -2$ with reason.

Solution:

The graph of f has a vertical asymptote at $x = -2$ because the denominator $2x^2 - 8$ equals 0 at $x = -2$ but the numerator does not equal 0 at $x = -2$.